

Determination of Cosmological Constant from Gauge Theory of Gravity

Ning Wu^{1,*}, Germano Resconi², Zhan Xu³, Zhi-Peng Zheng¹,
Da-Hua Zhang¹, Tu-Nan Ruan⁴

Institute of High Energy Physics, P.O.Box 918-1, Beijing 100039, P.R.China¹

Mathematical and Physical department, Universita Cattolica del Cacro Cuore,
Via Trieste 17, Brescia Italy²

Institute for Advanced Study, Tsinghua University, Beijing 100080, P.R.China³

Dep. of Modern Phys., University of Science and Technology of China,
Hefei, Anhui 230026, P.R.China⁴

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Abstract

Combining general relativity and gravitational gauge theory, the cosmological constant is determined theoretically. The cosmological constant is related to the average vacuum energy of gravitational gauge field. Because the vacuum energy of gravitational gauge field is negative, the cosmological constant is positive, which generates repulsive force on stars to make the expansion rate of the Universe accelerated. A rough estimation of it gives out its magnitude of the order of $10^{-52}m^{-2}$, which is well constant with experimental results.

*email address: wuning@ihep.ac.cn

1 Introduction

Cosmological constant is first introduced into physics by A.Einstein in 1917[1]. In that paper, he pointed that a small and positive cosmological constant is needed. At present, it is known that the cosmological constant has evoked much controversy in both cosmology and particle physics[2, 3]. Recent observations of high-redshift supernovae seem to suggest that the global geometry of the Universe may be affected by a small positive cosmological constant, which acts to accelerate the expansion rate with time[4, 5, 6, 7, 8].

Some theoretical models are proposed to explain the cosmological constant problems[9, 10]. In this paper, we will use another more fundamental way to determine the cosmological constant. Our analysis based on a new quantum theory of gravity — QUANTUM GAUGE THEORY OF GRAVITY[11, 12], which is formulated completely in the framework of quantum field theory and is based on action principle and gauge principle. Combining general relativity and quantum gauge theory of gravity, we can determine the cosmological constant in a simple way, which is well constant with experimental results. Finally, we will give a possible explanation on the possible origin of the cosmological constant.

2 Determination of the Cosmological Constant

First, let's discuss the Einstein's field equation from action principle. The action of the system is selected as[13]

$$A = A_E + A_M, \quad (2.1)$$

$$A_E = \frac{-1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\Lambda), \quad (2.2)$$

$$A_M = \int d^4x \mathcal{L}_M, \quad (2.3)$$

where G is the Newtonian gravitational constant, R is the scalar curvature, Λ is the cosmological constant, \mathcal{L}_M is the lagrangian density for matter fields, and

$$g = \det(g_{\mu\nu}), \quad (2.4)$$

with $g_{\mu\nu}$ is the space-time metric in general relativity. Using the following relations[13]

$$\delta\sqrt{-g} = \frac{1}{2}\sqrt{-g}g^{\mu\nu}\delta g_{\mu\nu}, \quad (2.5)$$

$$\sqrt{-g}g^{\mu\nu}\delta R_{\mu\nu} = \partial_\lambda W^\lambda, \quad (2.6)$$

$$T_m^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta A_M}{\delta g_{\mu\nu}(x)}, \quad (2.7)$$

where $T^{\mu\nu}$ is the energy-momentum tensor of matter fields and W^λ is a n contravariant vector, we can obtain the Einstein's field equation with cosmological constant Λ ,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}, \quad (2.8)$$

where $T_{\mu\nu}$ is the revised energy-momentum tensor, whose definition is

$$T_{\mu\nu} \triangleq T_{m\mu\nu} - \frac{1}{4\pi G} \frac{\delta\Lambda}{\delta g^{\mu\nu}} \quad (2.9)$$

Equation (2.1) is one of the starting point of our following discussions. Another starting point of our following discussions is from quantum gauge theory of gravity[11, 12]. In literature [11, 12], four different kinds of action for gravitational field are given. The best form for our present discussions is

$$A = \int d^4x \sqrt{-g} \mathcal{L}_0 + A_M, \quad (2.10)$$

where \mathcal{L}_0 is the lagrangian density for pure gravitational gauge field whose form is[11, 12]

$$\mathcal{L}_0 = -\frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma}g_{\alpha\beta}F_{\mu\nu}^\alpha F_{\rho\sigma}^\beta, \quad (2.11)$$

$F_{\mu\nu}^\alpha$ if the field strength of gravitational gauge field C_μ^α , which is given by

$$F_{\mu\nu}^\alpha = \partial_\mu C_\nu^\alpha - \partial_\nu C_\mu^\alpha - g C_\mu^\beta \partial_\beta C_\nu^\alpha + g C_\nu^\beta \partial_\beta C_\mu^\alpha. \quad (2.12)$$

Suppose that two actions eq.(2.1) and eq.(2.10) are essential the same, then we will get

$$\mathcal{L}_0 = \frac{-1}{16\pi G}(R - 2\Lambda). \quad (2.13)$$

It means the difference of two lagrangians of general relativity and quantum gauge theory of gravity gives out the cosmological constant

$$\Lambda = \frac{1}{2}(R + 4g^2\mathcal{L}_0). \quad (2.14)$$

In eq.(2.14), both scalar curvature R and lagrangian density \mathcal{L}_0 for pure gravitational gauge field are known, so, we can use eq.(2.14) to determine the cosmological constant Λ .

According to literature [11, 12], both scalar curvature R and lagrangian density \mathcal{L}_0 can be expressed by gravitational gauge field C_μ^α . So, the explicit expression of

$\Lambda(x)$ is

$$\begin{aligned}
\Lambda(x) = & 2g^{\mu\kappa}(\partial_\mu G \cdot G^{-1} \cdot \partial_\kappa G \cdot G^{-1})_\alpha^\alpha - g^{\mu\kappa}(\partial_\mu \partial_\kappa G \cdot G^{-1})_\alpha^\alpha \\
& + \frac{3}{4}\eta^{\rho\sigma}g_{\alpha\beta}g^{\mu\kappa}(\partial_\mu G)_\rho^\alpha(\partial_\kappa G)_\sigma^\beta - \eta^{\alpha\beta}G_\beta^\kappa(\partial_\kappa G \cdot G^{-1} \cdot \partial_\lambda G)_\alpha^\lambda \\
& - \eta^{\alpha\beta}G_\beta^\kappa(\partial_\lambda G \cdot G^{-1} \cdot \partial_\kappa G)_\alpha^\lambda + \eta^{\alpha\beta}G_\beta^\kappa(\partial_\kappa \partial_\lambda G)_\alpha^\lambda \\
& - \frac{5}{4}g_{\alpha\beta}\eta^{\rho\sigma}\eta^{\mu\nu}G_\nu^\kappa G_\rho^\lambda(\partial_\lambda G)_\mu^\alpha(\partial_\kappa G)_\sigma^\beta \\
& + \frac{1}{2}g_{\alpha\beta}\eta^{\alpha_1\beta_1}\eta^{\mu_1\nu_1}G_{\beta_1}^\nu G_{\mu_1}^\mu(\partial_\nu G)_{\alpha_1}^\alpha(\partial_\mu G)_{\nu_1}^\beta \\
& - g g^{\alpha\beta}F_{\alpha_1\beta_1}^\rho G_\rho^{-1\alpha_1}(G^{-1} \cdot \partial_\beta G \cdot G^{-1})_\alpha^{\beta_1} \\
& + \frac{1}{2}g\eta^{\alpha\mu}F_{\mu\beta}^\kappa(\partial_\kappa G \cdot G^{-1})_\alpha^\beta + \frac{1}{2}g\eta^{\alpha\mu}F_{\alpha\beta}^\rho G_\mu^\kappa(G^{-1} \cdot \partial_\kappa G \cdot G^{-1})_\rho^\beta \\
& + \frac{1}{2}g^2\eta^{\beta\beta_1}F_{\alpha\beta}^\rho F_{\alpha_1\beta_1}^{\rho_1}G_\rho^{-1\alpha}G_{\rho_1}^{-1\alpha_1} - \frac{g^2}{4}\eta^{\alpha\beta}F_{\mu\beta}^\rho F_{\alpha_1\alpha}^{\rho_1}G_\rho^{-1\alpha_1}G_{\rho_1}^{-1\mu} \\
& - \frac{3g^2}{4}g_{\alpha\beta}\eta^{\alpha_1\beta_1}\eta^{\mu\sigma}F_{\mu\beta_1}^\rho F_{\sigma\alpha_1}^\beta
\end{aligned} \tag{2.15}$$

From this equation, we can see that cosmological constant is space-time dependent, so we denoted it as $\Lambda(x)$. The cosmological constant which affects large scale structure evolution of cosmos is the average of $\Lambda(x)$ over the whole Universe.

3 Estimation of the Cosmological Constant

Now, let's make a rough estimation of the cosmological constant. Suppose that, the moving speed of stars is slow, so in the vacuum, the dominant component of gravitational field C_μ^α is C_0^0 and dominant field strength of gravitational gauge field is \vec{E} . Other components are much smaller than C_0^0 and \vec{E} , and they will be neglected in our estimation. In order to estimate the magnitude of Λ , we use our solar system as an example. Denote solar equatorial radius as r_0 and the distance between solar and nearest star as R_0 . The gravitational field C_0^0 which is generated by solar is [11, 12]

$$gC_0^0 = -\frac{GM}{rc^2}, \tag{3.1}$$

where r is the distance between a point in space and the sun. The only non-vanishing field strength \vec{E} is

$$g \vec{E} = -\frac{GM}{r^2c^2} \vec{r}. \tag{3.2}$$

In our estimation, the sun is regarded as quasi-static, so all time derivatives vanish. Using the equation of motion of pure gravitational gauge field in vacuum, the linear

terms in eq.(2.15) vanish[11, 12], so we only need to concern quadratic terms. Using all these relations, eq.(2.15) gives out the following relation

$$\Lambda(r) = 2 \left(\frac{GM}{r^2 c^2} \right)^2 + o(gC_\mu^\alpha)^3, \quad (r > r_0) \quad (3.3)$$

$$\Lambda(r) = 2 \left(\frac{GM}{r_0^3 c^2} \right)^2 \cdot r^2 + o(gC_\mu^\alpha)^3, \quad (r < r_0). \quad (3.4)$$

The average of $\Lambda(x)$ gives out cosmological constant,

$$\Lambda = \frac{\int_0^{R_0} \Lambda(r) 4\pi r^2 dr}{\int_0^{R_0} 4\pi r^2 dr} \simeq \frac{36 G^2 M^2}{5 R_0^3 r_0 c^4}. \quad (3.5)$$

The distance between the sun and the nearest star is about 4.5 light year, so

$$R_0 \sim 4.26 \times 10^{16} m. \quad (3.6)$$

The mass of the sun is

$$M = 1.99 \times 10^{30} kg, \quad (3.7)$$

and the radius of the sun is

$$r_0 = 6.96 \times 10^8 m. \quad (3.8)$$

Using all these relation, we get

$$\Lambda \sim 2.92 \times 10^{-52} m^{-2}. \quad (3.9)$$

This value is quite close to experimental results. According to PDG, the cosmological constant is[14]

$$\Lambda = 3.51 \times 10^{-52} \Omega_\Lambda h_0^2 m^{-2}, \quad (3.10)$$

with Ω_Λ is the scaled cosmological constant and h_0 is the normalized Hubble expansion rate, whose values are

$$-1 < \Omega_\Lambda < 2, \quad (3.11)$$

$$0.6 < h_0 < 0.8. \quad (3.12)$$

So, a quite rough estimation of cosmological constant is quite close to experimental value.

4 Comments

Because $-\left(\frac{GM}{r^2 c^2}\right)^2$ relates to the energy of gravitational field in vacuum, both space-time curvature and cosmological constant directly relate to the average energy of gravitational field in vacuum. In this paper, a very rough estimation is given. A

more strict estimation of cosmological constant should average $\Lambda(x)$ over the vacuum space of the whole Universe.

Because the energy of gravitational field in vacuum is negative, it generates a repulsive force on stars and makes the expansion rate of the Universe accelerated. This is the reason why cosmological constant is positive.

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References

- [1] A.Einstein, Sitz.Preuss.Akad.Wiss. 142-152, 1917. For an English translation, see **The Principle of Relativity** (Methuen, 1923, reprinted by Dover Publications), P.35.
- [2] S.Weinberg, " *The cosmological constant problems*", astro-ph/0005265.
- [3] S.M.Carroll, W.H.Press, E.L.Turner, Annu.Rev.Astron.Astrophys. **30**, 499(1992).
- [4] Idit Zehavi, Avishai Dekel, Nature **401**, 252 (1999)
- [5] J.P.Ostriker, Paul J. J.Steinhardt, Nature **377** 600 (1998)
- [6] S.Perlmutter, *et al.* Astrophys.J. **517** 565-586 (1999).
- [7] A.G.Riess, *et al.* Astron.J. **116** 1009-1038 (1998).
- [8] Limin Wang,R.R.Caldwell,J.P.Ostriker, and Paul J.Steinhardt, Astrophys.J. **530** 17-35 (2000)
- [9] R.Mignani, E.Pessa, G.Resconi, General Relativity and Gravitation, **29**, No. 8 1049 (1997)
- [10] Zhe Chang, Xin-Bing Hunag, "Matching the observed cosmological constant with vacuum energy density in Ads", astro-ph/0103437
- [11] Ning Wu, *Gauge Theory of Gravity*, hep-th/0109145;
- [12] Ning Wu, *Quantum Gauge Theory of Gravity* hep-th/0112062
- [13] Ren-Chuan Wang, "Introduction to General Relativity", (University of Science and Technology of China Press, 1996) P.198 (in chinese)
- [14] Particle Data Group, 2000, Euro. Phys. J. C15 1.